## T.3: Investigations on Higher Order Modes in RF Cavities of Accelerator

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### 1. Introduction

The recent developments in the field of particle accelerators have significantly increased the demand for higher energy acceleration. Therefore, in modern accelerators the major emphasis has been on increasing the voltage gradient across the RF (radio-frequency) cavity, on obtaining high magnetic field strength in magnets and on precise alignment for getting better luminosity and emittance of the beam at higher energies. The main limitations in achieving these goals come from technological constraints and different problems related to beam physics such as instabilities of various kinds. Many of these problems are related to RF cavities, namely multipacting, microphonics and Higher Order Modes (HOMs) [1, 2]. In most of these cases, the geometry of the RF cavity plays a role either directly or indirectly. For example, the multipacting of the cavity is related to the surface electric field which depends on the surface finish and cavity geometry. Similarly, the micophonics of the cavity can be related to (apart from external vibrations) the Lorentz electromagnetic pressure of the cavity which in turn depends on the electric and magnetic field distribution along the cavity inner profile. Among these, the problem of HOMs is an important issue that is related to both the accelerating beam and the RF cavity. This problem can be described as follows.

Fig. T.3.1 shows the acceleration of beam bunches in a circular accelerator using an RF cavity having a fundamental frequency $f_0$. The power from the RF source, at certain frequency (equal to $f_0$), is coupled to the cavity using a coupler. The beam bunch gets accelerated once it passes the cavity gap in right phase. While passing through the accelerating gap of the cavity, the various beam harmonics present in the accelerated beam (beam frequencies>$f_0$) may also resonate with and excite the higher frequency modes in the cavity. Unless these excited modes decay fast, they can affect the next bunch (trailing bunch) of particles which enter the accelerating gap. The harmful nature of any particular higher order mode depends on the electromagnetic parameters of that mode inside the RF cavity. Some HOMs of the cavity do not affect beam bunches even if their frequency coincides with the bunch frequency, as their coupling impedance is low and the effect of the HOM decays before the trailing bunch passes through the cavity gap. However, if the strength of the excitation (coupling impedance) of cavity HOMs and occurrence of resonance of any of these modes with the beam modes is significant, it can generate an instability known as HOM coupled bunch instability [3]. This instability may lead to beam current reduction or beam loss. In case of high beam current machines the beam breakup may lead to sensitizing (radioactivity) the cavity structure. Therefore, the higher order modes of the cavity should be such that either their (i.e., the HOMs') coupling coefficients are kept at low values or the resonance between the cavity HOMs and the beam harmonics are avoided.

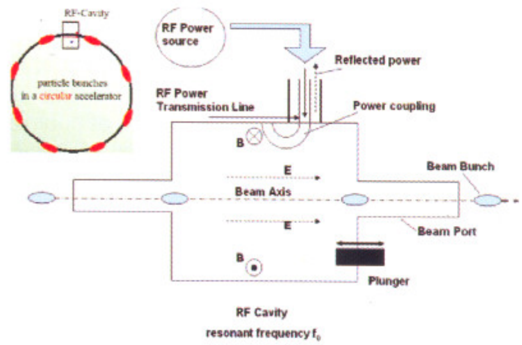


*Fig. T.3.1: Particle acceleration in an RF cavity of an accelerator*

The above HOM issue can be understood by taking an example of synchrotron accelerators. In synchrotron light sources (such as INDUS-2), HOMs become problematic due to close spacing between particle bunches being accelerated and the repetition of the same bunches again and again. Due to this, the beam bunch spectrum contains many frequencies and the chances of a resonance of these with the HOMs are high. If the synchrotron ring is filled with uniform equally spaced electron beam bunches (say B), the beam spectra will contain 'n' components of $Bf_{rev}$, where $f_{rev}$ is the revolution frequency of the particle in the synchrotron ring [4]. Considering the components of synchrotron oscillations, the total beam spectrum components are given as:

$$f_{\pm\mu n} = nBf_{rev} \pm (\mu f_{rev} + f_s) \quad (1)$$

where μ is an integer corresponding to coupled bunch mode number and $f_s$ is the frequency of synchrotron oscillation, which is given as

$$f_s = f_{rev}\sqrt{\frac{eVn\nu}{2\pi E_s}} \quad (2)$$

Here, V is the peak accelerating voltage, $E_s$ is the beam energy and μ the momentum compaction of synchrotron. The strength of the beam cavity interaction of HOMs depends on the frequency spacing of HOMs in the RF cavity (along with their longitudinal and transverse coupling impedances) with respect to beam harmonics (which is different at different beam energies as shown in the above equation). A strong resonance of cavity HOMs with beam frequencies may lead to beam loss.

### 1.1 Brief Literature review about HOM mitigation techniques

The problem of HOMs in any accelerator is generally mitigated either by changing the HOM spectrum or by damping the HOMs. Changing the HOM spectrum is carried out by changing the geometry (profile) or volume of the resonant cavity. These methods are very effectively used in various electron synchrotrons such as Elletra, ANKA, INDUS-2 [5], SLS booster and storage ring (499.65 MHz), LNLS (Laboratorio Nacional de Luz Sincrotron, Brazil) booster and storage ring (476 MHz) etc. The experimental investigations by M Svandrlik and group at Elettra [6] suggests that temperature tuning of the storage ring cavities has been effective for shifting the dangerous cavity HOMs for third generation light sources.

Another method to reduce HOM instabilities is to damp the HOMs by coupling the fields induced by the beam at HOM frequencies to some external loads and not allowing them to build up to a level where these can affect the beam. The beam current thresholds for the excitation of coupled bunch instabilities are inversely proportional to the total HOM impedance $Z \sim (R_{sh}/Q_0)HOMQ_{ext}$ Thus one has to minimize the $R/Q_0$ and the external quality factor ($Q_{ext}$) of the HOMs. The HOMs with frequencies above the beam pipe cut-off frequency propagate down the vacuum chamber and are damped by the resistance of the chamber material [7]. Different designs of coupling probes or antennae, loops and waveguides are used as HOM couplers at different places. However, a completely HOM damped cavity is yet to be evolved.

### 1.2 Objective of HOM study

It is apparent from the previous discussion that one cannot avoid HOMs in RF cavities. However, it has been pointed out that not all HOMs are problematic and a particular HOM can potentially create problems only if its coupling impedance is high. Keeping this in mind, in the present investigations, the solutions are searched that lead to cavity designs with a minimum number of significant HOMs (i.e., those that can create problems). Of course the solutions have to ensure that the fundamental mode does not suffer. Again, the success lies not in ensuring absolutely no significant HOMs (this would indeed be a remarkable success), but in reducing the significant HOMs to manageable numbers, say three or four.

To achieve a design with fewer significant HOMs, two methods are attempted: a semi-analytical one and a parametric one. The semi-analytical approach is carried out by recognizing that most accelerating cavities are ellipsoidal in shape and can be easily approximated using the oblate spheroidal shape, where it is possible to obtain the analytical solution by solving the wave equation in spheroidal coordinates. Solutions to the wave equation will lead to the eigenvalues of the cavity which are nothing but the fundamental and higher order modes. Once such solutions are obtained, one can study which parameters may lead to shapes with fewer significant HOMs. As a further step to achieving this goal, a parametric approach has been attempted that does not restrict itself to ellipsoidal shapes but looks at the entire gamut of shapes by parametrizing the cavity into different components. To understand HOMs, initially, the study on an existing cavity of INDUS-2 accelerator is performed and found how the existing mechanisms of changing cooling water temperature and HOM frequency shifter help to shift the HOMs.

This has led to three significant contributions to reduce the HOM problem in the accelerating cavity. These are taken up one by one as follows.

### 2. Estimation, observation and mitigation study of the HOMs in INDUS-2

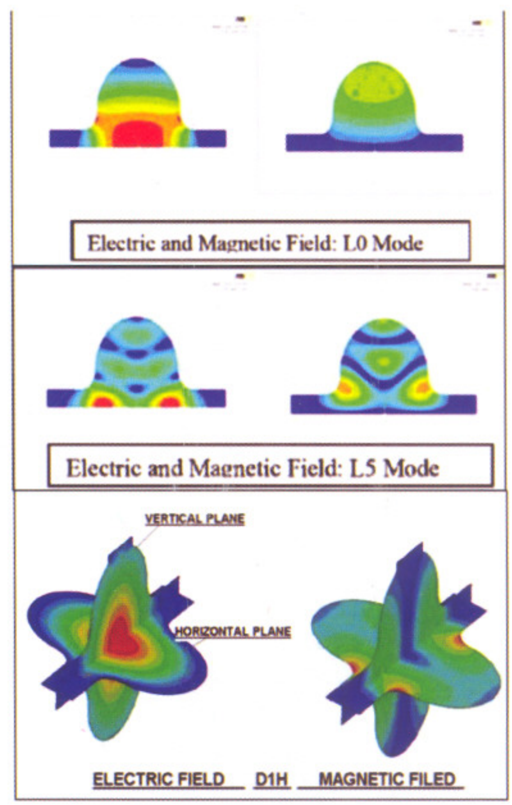


*Fig. T.3.2: (a) Resultant Electric (E) and Magnetic field (H) plot of L0 accelerating mode (top), (b)L5 HOM mode (middle) on the RF cavity and (c)D1H HOM mode on the RF cavity. The E field values vary between 0 (blue) and 1 V/m (red) and corresponding H field values vary between 0 and 0.0024 A/m$^2$.*

INDUS-2 cavities are equipped with two independent mechanisms to avoid HOMs; 1. The precision temperature control system of the cooling system and, 2. Higher Order Mode Frequency Shifter (HOMFS). Initially, 3D electromagnetic simulations of the INDUS-2 cavity have been carried out for estimation of electromagnetic parameters of HOMs such as their frequencies, their E and H fields, quality factors, shunt impedances and transit time factors. Fig. T.3.2 shows an example of electric and magnetic (E and H) for accelerating mode L0, one of the longitudinal HOM L5 and one of the dipole modes HOM D1H (1st dipole mode with horizontal component). In the next step, coupled electromagnetic-thermal-structural simulations (depicted in Fig. T.3.3(a)) have been carried out to obtain accurate information regarding HOMs in different scenarios. A 1/12th symmetric combined model of vacuum and cavity zones as shown in Fig. T.3.3(b) is taken for coupled simulations. The sensitivity of HOMs, with respect to operating conditions such as inlet temperature of coolant as shown in Fig. T.3.4(a) (from 35°C to 85°C), HOMFS plunger position as shown in Fig. T.3.4(b) (from 0 to 50 mm) and operating cavity voltage or corresponding heat dissipation (upto 60 kW heat load on cavity surface), has been estimated using this combined methodology (coupled electromagnetic-thermal-structural simulations). Some of these results in Fig. T.3.5 (a) and (b) illustrates that temperature change of inlet coolant or change of position of plunger changes all HOMs differently. A detailed data bank of HOMs for various possible operating scenarios is generated and used to investigate safe operating scenarios.

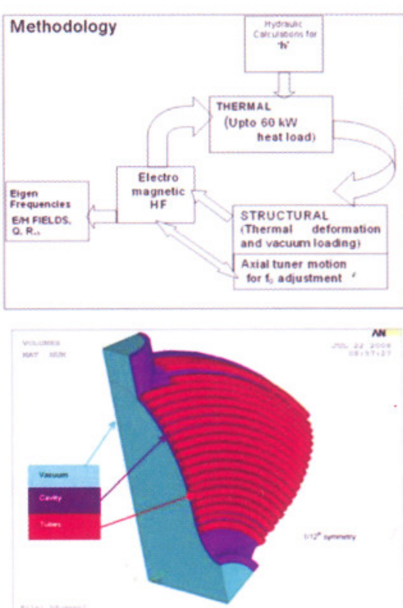


*Fig. T.3.3: (a) Coupled sequence cycle for INDUS-2 cavity calculations (top). (b) 1/12th Combined Model of INDUS-2 used for couple analysis (bottom)*

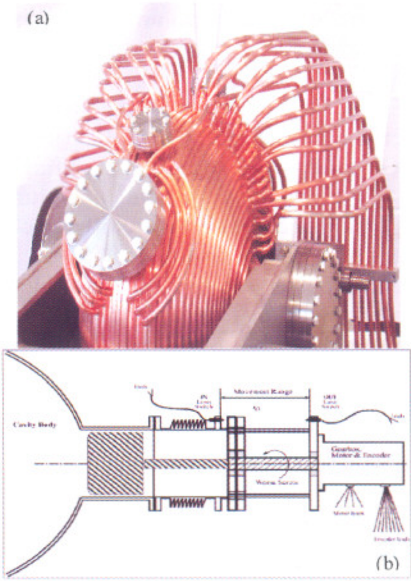


*Fig. T.3.4: (a) Means of HOM shifting by varying cooling water temperature from 35°C to 85°C (top) and (b) Plunger tuner from 0 to 50 mm (bottom).*

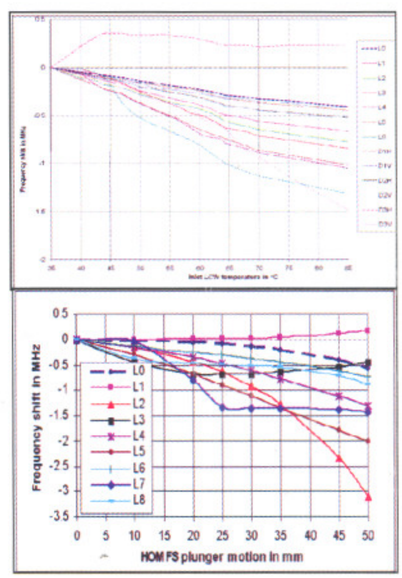


*Fig. T.3.5: Results of the effect of inlet temperature change from 35°C to 85°C (top) and Plunge position change 0 mm to 50 mm (bottom) on different HOMs.*

Safe HOM-free operating windows, using a combination of cavity inlet temperature, plunger position and power dissipation for each unsafe HOM have been worked out at various operating conditions of the INDUS-2 machine using a concept of the matching cavity HOMs and beam frequencies. In order to find the resonance between beam frequencies and cavity HOMs, a frequency ratio is defined which is named as matching index (MI). The MI is given as is the difference in the

$$MI = \frac{f_d}{(BW)_{HOM}} \quad (3)$$

Where $f_d = f_{HOM} - f_{\pm\mu n}$

frequency of an HOM and the nearest beam spectrum and (BW)HOM is the resonance band width of any HOM.

The index MI signifies that if its value is less than 1, there is definitely a resonance between a beam frequency and that particular HOM frequency. Three zones are defined for this index, which are as follows:

1. HOM resonance zone if MI <1 i.e. HOM is resonating with the beam frequency.
2. HOM probable zone for 1 <MI < 2 i.e. HOM bandwidth and the beam frequency do not match, but only by a narrow margin.
3. MI >2 is considered safe zone. HOM and beam frequencies are away from each other with sufficient margin.

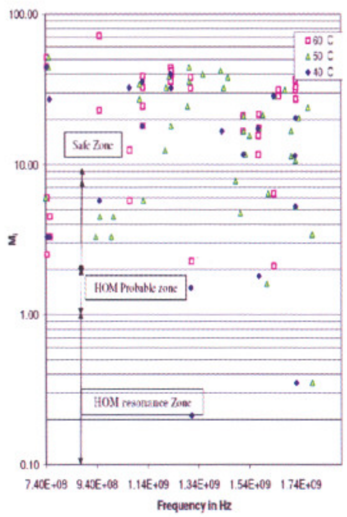


*Fig. T.3.6: Matching index for 40°C, 50°C and 60 °C of coolant water temperature and 50 mm plunger position for power dissipation of 20 kW.*

Based on this concept, a program is written to collect only safe operating scenario data. The safe operating coolant temperature ranges and plunger positions for power dissipation of 20 kW, 40 kW and 60 kW is worked out. Fig. T.3.6 illustrates the comparison of the Matching Indices for different coolant inlet temperatures at a fixed plunger position of 50 mm and a power dissipation of 20 kW on the cavity walls. In this case, an inlet temperature of 60°C can be termed as HOM free operating situation.

### 3. Analytical study of oblate spheroidal RF cavity for HOMs

After devising methods to avoid HOMs in regular cavities, in the next step, a semi analytical approach is adopted to understand and solve the HOM issue right from the design stage by using the oblate spheroidal shape [8]. The oblate spheroidal shape is close to the common elliptical RF cavity shape used in accelerators. These shapes can be defined in the spheroidal coordinate $\xi,\eta,\phi$ The value of $\xi$ defines the outer boundary of the oblate spheroid while the parameter $\eta$ describes a system of hyperbolas and the parameter $\phi$ describes a plane. The analytical solution of the oblate spheroidal cavities is worked out by separating the spheroidal wave equation into angular and radial parts using separation of variables [9]. The resulting solutions of the spheroidal radial and angular wave functions yield the eigen-frequencies. A plot of the derivative of radial functions which is used to find for TM mode frequencies is shown in Fig. T.3.7(a). The TM eigen-frequencies are obtained from the zeros of this function.

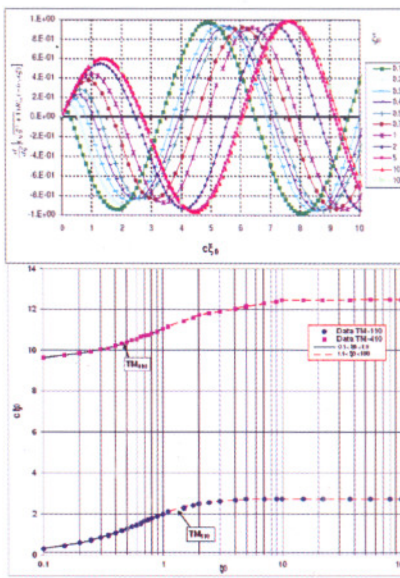

*Fig. T.3.7: (a) Plot of the function of derivative of spheroidal radial wave function TM modes are plotted against $c\xi_0$ by varying $\xi_0$ from 0.1 to 100, (b) Frequencies in terms of $c\xi_0$ Vs. $\xi_0$ for TM modes.*

As an example, the exactly calculated frequencies (in terms of $c\xi_0$ of two TM modes (110 and 140) are plotted as a function of $\xi_0$ in Fig. T.3.7(b) for $\xi_0$ ranging from 0.1 to 100. It can be seen that the curves shown in Fig. T.3.7(b) have a high degree of nonlinearity in the entire range of $\xi_0$ = 0.1 to 100. Since a single curve fit would have been poor in this entire range, the range for $\xi_0$ is divided into $\xi_0$ = 0.1 to 1 and $\xi_0$ 1.01 to 100 for the purpose of fitting. The formulation of this curve-fit for the lower range of ξ is given by a polynomial equation (Eq.4)

$$f_{mn0} = \frac{1}{\pi d\xi\sqrt{\mu\varepsilon}}\left[a_6\xi^6 + a_5\xi^5 + a_4\xi^4 + a_3\xi^3 + a_2\xi^2 + a_1\xi + a_0\right]$$

$$+ a_3\xi^3 + a_2\xi^2 + a_1\xi + a_0] \qquad (4)$$

within the range ($0.1 < \xi_0 < 1$).

The coefficients $a_6$ to $a_0$ in the above equation are calculated for each TM and TE mode for the range $\xi_0 = 0.1$ to 1. Though such a curve-fit has been obtained for the range $\xi_0 = 1.1$ to 100, these are not practical since all cavities have $\xi_0 < 1$.

Similarly the radial and angular functions are also fitted in terms of $\xi_0$ and in terms of η and the expressions for the magnetic and electric fields are obtained. As an example, the expression for the magnetic and electric fields for $TM_{110}$ mode can be given as follows:

$$H_\phi = \frac{A_n}{h_\phi}(-0.1783\xi^2 + 0.144\xi + 0.5)\sqrt{(\xi^2+1)}$$

$$(-1.4359\eta^2 + 0.5954\eta + 0.949)\sqrt{(1-\eta^2)} \qquad (5)$$

$$E_\xi = \frac{A_n(-0.1783\xi^2 + 0.144\xi + 0.5)\sqrt{(\xi^2+1))}}{j\omega h_\eta h_\phi} -$$

$$\frac{\partial}{\partial\eta}\left(\left(-1.4359\eta^2 + 0.5954\eta + 0.949\right)\sqrt{(1-\eta^2)}\right) \qquad (6)$$

$$E_\eta = -\frac{A_n(-1.4359\eta^2 + 0.5954\eta + 0.949)\sqrt{(1-\eta^2)}}{j\omega h_\xi h_\phi} -$$

$$\frac{\partial}{\partial\xi}\left(\left(-0.1783\xi^2 + 0.144\xi + 0.5\right)\sqrt{(\xi^2+1)}\right) \qquad (7)$$

where $A_n$ is a constant and $h_\xi$, $h_\eta$ and $h_\phi$ are the metric coefficients, which are functions of ξ and η and φ.

Using the above field equations (Eqs. 5-7) in oblate spheroidal coordinates, one can obtain the characteristic constants of an RF cavity such as transit time factor, quality factor and shunt impedance. The validity of these calculations for different TM modes are checked with the corresponding values for an equivalent oblate spheroidal shape of the INDUS-2 cavity. The agreement in the values of both the frequencies of the HOMs as well as the corresponding characteristic constants is found to be very good. Decreasing the value of ξ of the oblate cavity decreases the value of TM and TE mode frequencies in the spectrum, hence smaller ξ values have narrow frequency spacing in the spectrum.

To investigate this further, Fig. T.3.8 plots a characteristic parameter ratio denoted as $S_{HOM}$ (a ratio of $(R_{sh}/Q_0)_{HOM}$ to $(R_{sh}/Q_0)_{f0}$) of all HOMs up till the $TM_{420}$ mode (which is generally above the beam cut-off frequency), over the range of $0.1 < \xi > 1$. An HOM is considered to be significant if $S_{HOM}$ (a ratio of $R_{sh}/Q_0$ for HOM and fundamental frequency) exceeded 0.1. A higher value of $S_{HOM}$ indicates a longer decay time, and the value of 0.1 was chosen such that the effect of HOM dies down before the next bunch enters the cavity. It is observed from the figure that there are only 2 significant TM HOMs in the range ξ = 0.69 to 0.77. For the ranges ξ< 0.69 and ξ > 0.77, there are at least 3 or more significant TM HOMs. In this range of optimal ξ other parameters such as quality factor, shunt impedance, transit time factor and peak magnetic and electric field ratios are found suitable as far as the accelerating properties are concerned. Thus, it is predicted that cavities with ξ in the range 0.69-0.77 will perform better. Hence for any cavity design the starting point for the ξ value can be taken near 0.7. Subsequently all the other cavity characteristic parameters can be examined. This is consistent with the experimental evidence that typical RF cavities are indeed in this range like 1.3 GHz ILC or Tesla cavities (ξ = 0.66).

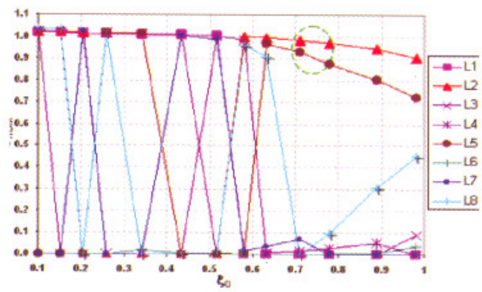


*Fig. T.3.8: Effect of ξ on $S_{HOM}$ for TM HOMs*

**4. Investigation of various cavity profile parameters using parametric approach**

The investigation of design of the cavity is now carried further by allowing the shape to deviates from the ellipsoidal shape. This is done using a parametric methodology that studies the effect of different cavity dimensions such as cavity radius $C_r$, beam pipe radius $C_b$, equator ellipse dimensions $a_e$, $b_e$, iris ellipse dimensions $a_i$, bi and wall angle α (as shown in Fig. T.3.9(a)). By varying all the parameters, various shapes were obtained and all important characteristic parameters of the resulting HOMs were calculated. The shape with the least number of HOMs was termed as optimized Model or Model-O (as shown in Fig. T.3.9(b). A comparison of INDUS-2 and Model-O cavity HOM data shows a tremendous drop in significant HOMs from 16 to 3 [10]. This is the great advantage of the optimized shape that now it is needed to control only these 3 HOMs. This profile gives another advantage of finding many safe operating scenarios using the Matching Index (MI) concept.

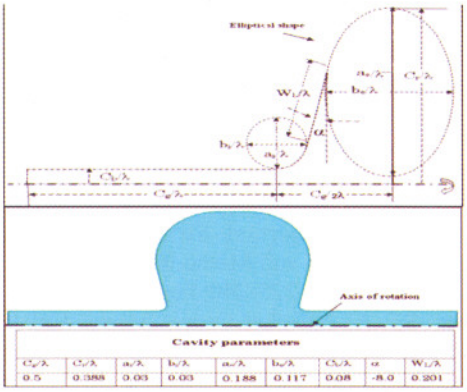


*Fig. T.3.9: (a) Parametric dimensions of RF cavity shown by half cylindrically symmetric profile. (b) Optimal profile of the cavity for least number of significant HOMs*

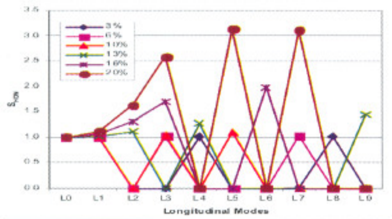


*Fig. T.3.10: Effect of increasing iris radius on the characteristic constant (SHOM) for different longitudinal modes.*

It is reviled from the parametric analysis that the iris curvature dimension is one of the most sensitive dimensions for HOMs. The iris curvature is defined by an ellipse of dimensions ai and bi (as shown in Fig. T.3.9). The values of SHOM increase with iris curvature for most longitudinal HOMs. This effect is depicted in Fig. T.3.10 for six different increments of the iris curvature in percentage value (with respect to the cavity gap).

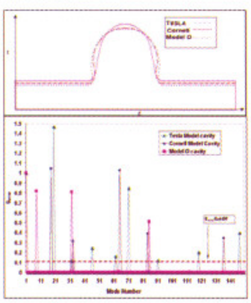

*Fig. T.3.11: Tesla, Cornell and Model O shapes of single cell RF cavities for a fundamental frequency of 1.3 GHz (top) and SHOM of first 150 modes (bottom).*

Further, this profile is evaluated for a fundamental frequency of 1.3GHz corresponding to the ILC superconducting cavity [11]. This profile leads to a cavity shape with not only low number of significant HOMs but also reduced loss characteristics. A comparison of the cavity profile and SHOM of the low loss Cornell cavity, the Tesla cavity and the Model-O cavity is illustrated in Fig. T.3.11.

The values of various characteristic parameters of the fundamental frequency are calculated for all these shapes. The values of the beam pipe radius, $H_{pk}/E_{acc}$, $E_{pk}/E_{acc}$, and $R_{sh}/Q_0$ are tabulated in Table T.3.1. The values of the characteristic parameters, $H_{pk}/E_{acc}$ and $E_{pk}/E_{acc}$ obtained by Shemelin [12] are also included for comparison in Table T.3.1 (in bold font and within brackets). The agreement is excellent, and serves as a check for the correctness of calculations.

*Table T.3.1 Comparison of cavity characteristics for the fundamental frequency. The figures in the brackets are those from reported data of the Tesla and Cornell Profiles.*

| Parameters | Tesla cavity Model /[reported] | Cornell Low loss Cavity Model /[reported] | HOM reduced cavity Model-o |
|---|---|---|---|
| $C_b$ | 70 mm | 60 mm | 52 mm |
| $E_{pk}/E_{acc}$ | 1.97[**2.0**] | 2.24 [**2.22**] | 2.41 |
| $H_{pk}/E_{acc}$ (Oe/MV/m) | 41.6 [**42**] | 37.1[ **37.6**] | 36.3 |
| $R_{sh}/Q_0$ | 56.82 | 63.39 | 67.67 |

For superconducting cavities, it is found that the optimal shape (Model O) is similar to the Cornell reentrant cavity, but performs even better. The loss is slightly lower, but the number of significant HOMs decreases substantially, from five to three. Thus, the Model O superconducting cavity can be considered a significant improvement. The only problem is the slightly (10%) higher value of $E_{pk}/E_{acc}$ which enhance the chances of multipacting. However, with the better surface treatment processes available nowadays, this problem can be dealt with. Therefore it is suggested the use of this shape for high-current applications such as energy-recovery linacs and synchrotrons. This work can be extended to multi-cell superconducting RF cavities.

## 5. Conclusion

In summary, the research work presents three important contributions towards the mitigation of the HOM problem in RF accelerating cavities:

1. A safe (HOM free) operating window, using a combination of cavity inlet temperature, plunger position and power dissipation for each unsafe HOM is worked out at various operating conditions of the INDUS-2

machine using the matching index (MI) concept. The operating zones for MI >2 are suggested for safe or HOM free operations. This concept will definitely be helpful for smooth operation of INDUS-2. This concept can also be implemented in other kind of accelerating systems for avoiding HOMs.

2. An analytical formulation for ellipsoidal cavities is obtained for the first time. Further, an efficient range for the cavity parameter $\xi$ is calculated, which can be used to design ellipsoidal cavities with reduced number of HOM problems.
3. A low loss cavity with few significant HOMs is obtained by carrying out a parametric study of RF cavities by keeping the fundamental frequency of the cavity constant. This is a significant improvement over the existing design and is ideal for high current applications such as ERLs, synchrotrons and Linacs.

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