2D Detectors for Powder X-ray Diffraction: Data Reduction and Rietveld Refinements

Anthony Arulraj Ramanujan Fellow MSG, IGCAR



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E6 Focusing diffractometer at BENSC



Comparison can be made using the following technical details: Detective Quantum Effciency (DQE), Point spread function, size, speed, dynamic range

Ref. Powder Diffraction: Theory and Practice

Types of Area Detectors

22 20 (°)

Detector types Comparison

Technical specifications Number of modules $3 \times 8 = 24$ Sensor Reverse-biased silicon diode array Sensor thickness 320 µm Pixel size $172 \times 172 \mu m^2$ Format 1475 × 1679 = 2,476,525 pixels Area 254 × 289 mm2 Intermodule gap x: 7 pixels, y: 17 pixels, 8.0 % of total area Dynamic range 20 bits (1:1,048,576) Counting rate per pixel > 2 x 10⁶ Xray/s Energy range 3 – 30 keV Quantum efficiency 3 keV: 80 % (calculated) 8 keV: 99 % 15 keV: 55 % Energy resolution 500 eV Adjustable threshold range 2 – 20 keV Threshold dispersion 50 eV Readout time 2.2 ms Framing rate 30 Hz Point-spread function 1 pixel

Ref. Powder Diffraction: Theory and Practice

Data Analysis

- •Calibration and determination of the Detector Characteristic
- •Determination of the experimental geometry (Beam Center, tilt)
- Application of the data reduction procedure
- •Application of the diffraction geometry related corrections

O.K. CANCEL	? HELP	INFO
DESCRIPTIONS	VALUES	CHANGE
SAMPLE TO DETECTOR DISTANCE (MM) (STARING)	120.0000	DISTANCE
WAVELENGTH (ANGSTROMS) (STARTING)	0.652863	WAVELENGTH
SIZE OF HORIZONTAL FIXELS (MICRONS)	100.0000	X-PIXEL SIZE
SIZE OF VERIICAL PIXELS (MICRONS)	100.0000	Y-FIXEL SIZE
NUMBER OF AZIMUTHAL SECTIONS	180	ANGULAR SECTIONS
REFECT OUT-LYING POSITIONS AND RE-REFINE	YES	REJECT OUTLIERS
reject limit from ideal (standard deviations)	2.000000	REFECT HIMIT
OUTPUT FULL INFORMATION	YES	FULLINFO
REFINE X/Y BEAM CENTRE	YES	REFINE BEAM X/Y
REFINE SAMPLE TO DETECTOR DISTANCE	YES	REFINE DISTANCE
REFINE X-RAY WAVELENGTH	YES	REFINE WAVELENGTH
REFINE DETECTOR NON-ORTHOGONALITY	YES	REFINE TILT
FIT INTERMEDIATE NUMBER OF RINGS	NO	EXTRAITERATIONS

LaB₆ for sample-detector distance, tilt etc.

O.K. CANCEL	? HELP	INFO
DESCRIPTIONS	VALUES	CHANGE
SIZE OF HORIZONTAL PIXELS (MICRONS)	100.0000	X-PIXEL SIZE
SIZE OF VERTICAL PIXELS (MICRONS)	100.0000	Y-PIXEL SIZE
SAMPLE TO DETECTOR DISTANCE (MM)	119.7349	DISTANCE
WAVELENGTH (ANGSTROMS)	0.652721	WAVELENGTH
X-PIXEL COORDINATE OF DIRECT BEAM	1734.759	X-BEAM CENTRE
Y-PIXEL COORDINATE OF DIRECT BEAM	1727.645	Y-BEAM CENTRE
ROTATION ANGLE OF TILTING PLANE (DEGREES)	33.93871	TILT ROTATION
ANGLE OF DETECTOR TILT IN PLANE (DEGREES)	0.284901	ANGLE OF TILT

LaB₆ for sample-detector distance: Refined Value

O.K. CANCEL	? HELP	INFO
DESCRIPTIONS	VALUES	CHANGE
STARTING AZIMUTH ANGLE (DEGREES)	0.0	START AZIMUTH
END AZIMUTH ANGLE (DEGREES)	360.0000	END AZIMUTH
inner radial limit (Pixels)	0.0	INNER RADIUS
OUTER RADIAL LIMIT (PIXELS)	1712.064	OUTER RADIUS
SCAN TYPE (RADIAL, 2-THETA, Q-SPACE)	2-THETA	SCAN TYPE
DEFAULT TO APPROX. 1 DEGREE SIZE AZIMUTHAL BINS	NO	1 DEGREE AZ
NUMBER OF AZIMUTHAL BINS	360	AZIMOTH BINS
NUMBER OF RADIAL/2-THETA BINS	1712	RADIAL BINS
INTENSITY CONSERVATION	NO	CONSERVEINT.
Apply Polanisation Correction	YES	POLANSATION
POLARISATION FACTOR	0.990000	FACTOR
MAXIMUM FOR D-SPACINGS SCANS (ANGSTROMS)	20.00000	MAX. D-SPACING
GEOMETRICAL CORRECTION TO INTENSITIES	YES	GEOMETRY COR.

O.K. CANCEL	? HELP	INFO
DESCRIPTIONS	VALUES	CHANGE
SCAN TYPE (D, RADIAL, 2-THETA, Q-SPACE)	2-THETA	SCAN TYPE
INTENSITY CONSERVATION	NO	CONSERVE INI.
APPLY POLARISATION CORRECTION	YES	POLARISATION
POLARISATION FACTOR	0.990000	PACTOR
GEOMETRICAL CORRECTION TO INTENSITIES	YES	GEOMETRY COR.
MAXIMUM 2-THETA ANGLE OF SCAN (DEGREES)	65.00000	MAX. ANGLE
NUMBER OF BINS IN OUTPUT SCAN	3450	SCAN EINS
MAXIMUM FOR D-SPACINGS SCANS (ANGSTROMS)	20.00000	MAX. D-SPACING

Data Reduction using Fit2d Polarization Corrections Lorenze correction Absorption correction Polarization correction(Fit2D) = $(I_h - I_v)/(I_h+I_v)$ I_h----> horizontal component (direction of X-rays) T_v----> Vertical component

Relationship between a pixel position and the solid angle covered by it.

Geometrical corrections to Intensities $\frac{F(x, y)}{F(0, 0)} = \frac{D^3}{R^3} = \frac{D^3}{(D^2 + x^2 + y^2)^{3/2}} = \cos^3 \varphi$

Angular range covered by the pixel in y and x directions

$$\Delta \beta = \frac{D}{D^2 + y^2} \Delta y = \frac{D}{{R'}^2} \Delta y \qquad \Delta \alpha = \frac{\Delta x}{R'}$$

Solid Angle covered by the pixel

$$\Delta \Omega = \Delta \alpha \cdot \Delta \beta = \frac{D}{R'^3} \Delta y \cdot \Delta x = \frac{D}{R'^3} \Delta A$$

For a pixel at arbitrary position P(x,y)

$$R = \sqrt{D^2 + x^2 + y^2} = \sqrt{D^2 + r^2}$$

Flux at P(x,y)

$$F(x, y) = \Delta \Omega B = \frac{\Delta ADB}{R^3} = \frac{\Delta ADB}{(D^2 + x^2 + y^2)^{3/2}}$$

Flux in photons/s and brightness of S in photons/s per mrad²

Figure 1. a) 2D image from the Mar345 Image Plate detector set perpendicular to the incident X-ray beam. b) Intensity versus 2 θ plot obtained from the 2D image after various data reduction procedures as implemented in the program FIT2D. In the inset an expanded region of the plot is shown (λ =0.6519 Å; sample to detector distance=125.22 mm.)

Figure 2. a) 2D image from the Mar345 Image Plate detector tilted w.r.t. the incident X-ray beam (angle of tilt = 30 deg). b) Intensity versus 2 θ plot obtained from the 2D image using the program FIT2D. In the inset an expanded region of the plot is shown (λ =0.6519 Å; sample to detector distance=305 mm.)

Figure 3. a) Rietveld refinement data for the $GdBaCo_2O_{5.5}$ system. The plot shows the observed and calculated Intensity versus 20 values. The difference plot is also shown. In the inset an expanded region of the plot is shown. b) The crystal structure of $GdBaCo_2O_{5.5}$ from the refined values of lattice and structural parameters obtained from the Rietveld refinement. Neutron Powder Diffraction

(Ti, yi), i= 1,n

 $T_i \rightarrow scattering variable$; 20, TOF, Energy

 $y_i \rightarrow intensity$

Anthony Arulraj Tel.: 030 8062-2793 | Fax: 030 8062-2999 anthony.arulraj@helmholtz-berlin.de

The Rietveld Method

The Rietveld method is a structure refinement method that consists of refining a crystal (and/or magnetic) structure by minimizing the weighted squared difference between the observed and calculated pattern.

$$s_{y} = \sum_{i}^{n} W_{i} (y_{i} - y_{ci})^{2}$$
$$W_{i} = \frac{1}{y_{i}}$$

- y_i = observed intensity at the ith step,
- y_{ci} = calculated intensity at the ith step,

 \mathcal{N} = total number of steps.

Anthony Arulraj Tel.: 030 8062-2793 | Fax: 030 8062-2999 anthony.arulraj@helmholtz-berlin.de

The Rietveld Method

$$y_{ci} = s \sum_{K} L_{K} |F_{K}|^{2} \varphi (2\Theta_{i} - 2\Theta_{K}) P_{K} A + y_{bi}$$

 $S \rightarrow \text{Scale factor}$

 $K \rightarrow$ miller indices h, k, l

 $\mathrm{F}_{\mathrm{K}} \rightarrow \mathrm{structure} \ \mathrm{factor}$

 $L_K \rightarrow$ includes Lorentz factor, multiplicity

 $\phi \! \rightarrow \! \text{profile}$ shape function

 $P_K \rightarrow$ preferred orientation

 $A \rightarrow Absorption \ correction$

 $y_{bi} \rightarrow background$ intensity

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Structure Factors

$$F_{hkl} = \sum f_i \exp\left(-B_i \left(\frac{\sin\theta}{\lambda}\right)^2\right) \exp\left[2\pi i \left(hx_i + ky_i + lz_i\right)\right]$$
$$F_{mag} = \sum q_i p_i \exp\left[2\pi i \left(hx_i + ky_i + lz_i\right)\right]$$

 $\begin{array}{c} p_i \rightarrow magnitude \ of \ magnetic \ cross \ section \\ p=0.54Sf_m \! \times \! 10^{\text{-12}} \ cm \end{array}$

 $S \rightarrow magnetic moment$

 $f_m \rightarrow magnetic \text{ form factor}$

 $q_i \rightarrow magnetic interaction$ vector $q_j = \varepsilon (\varepsilon \cdot K_j) - K_j$

magnetic interaction vector for the jth atom depends on the unit vector (ϵ) for the scattering vector (**h**) and the unit vector for the magnetic moment on that atom(**K**j)

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Numerical criteria of the fit

$$R_{wp} = \begin{bmatrix} \frac{\sum w_i (y_i - y_{ci})^2}{i} \\ \frac{\sum w_i (y_i)^2}{i} \end{bmatrix} \qquad \qquad R_{exp} = \begin{bmatrix} \frac{n - P}{\sum w_i (y_i)^2} \\ \frac{1}{i} \end{bmatrix}$$

$$\chi^2 = \left[\frac{R_{wp}}{R_{exp}}\right]$$

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Two sets of refinable parameters

Profile parameters:

•Peak positions: A, B, C....; Zero shift

$$\frac{1}{d^{2}} = A.h^{2} + B.k^{2} + C.l^{2} + D.k.l + E.h.l + F.h.k$$

•Profile: U, V, W etc...

Structural parameters:

Scale factor

•Atomic coordinates: x_i, y_i, z_i

•Temperature parameter: B_i

•Components of the magnetic moment vector: K_i

Anthony Arulraj Tel.: 030 8062-2793 | Fax: 030 8062-2999 anthony.arulraj@helmholtz-berlin.de

LeBail fit

Anthony Arulraj Tel.: 030 8062-2793 | Fax: 030 8062-2999 anthony.arulraj@helmholtz-berlin.de

Rietveld refinement

Anthony Arulraj Tel.: 030 8062-2793 | Fax: 030 8062-2999 anthony.arulraj@helmholtz-berlin.de

THANK YOU