X-ray diffraction: A tool for Materials Research

Structure and Properties of functional materials

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(Out lines)

- Functional Materials
- Rietveld refinement
- Selected examples of structure and properties

- Framework solids
- Perovskite and related materials
- Dilute magnetic semiconductor
- Others





Energy Related Materials

Chemistry Division

Polymers

Nano-Materials





Thermal expansion

- Electrical and Magnetic properties
- Compound with fission products
- High pressure/High temperature effects
- Novel and unusual compounds

Preparation methods

Ceramic method	Solid State Synthesis	
	Combustion synthesis,	
Soft-chemical	Template method	
methods	Coprecipitation	
	Polyol method,	
	Sono-chemical	
	Hydro & solvothermal methods	
	Xero-gel method.	
	Vacuum heat treatment	
Other methods	Melt and quench technique	
	Flux method	
	High pressure synthesis	

Data collection strategy

Just for phase identification:

10° to 90°; Step size 0.02°, time per step 0.5 to 1 sec

For structural work:

5° to 110°; Step size 0.02°, time per step 3 to 10 sec

Selection of step size depends on the required resolution:

A peak of 0.3° FWHM can be nicely constructed with about 10 to 15 data points (step size 0.02°)

What we normally expect from the diffraction studies

Accurate unit cell parameter and symmetry Accurate structural parameters Identification of segregated secondary phase Correlation with physical properties

Choice of x-ray source

Accurate unit cell

Wavelength (λ)

Intensity

Resolutions

Completeness of data

Weak peaks

S/N ratio

Separation of closely spaced peaks

Symmetry

Good profile shape

Identifications of merged reflections

Stable refinement

Accurate structural parameters

Comparison of PXRD data of different sources

	SEALED TUBE	RAG	SR
Wavelength	Fixed (K $\alpha_{1and 2}$)	Fixed (K $\alpha_{1and 2}$)	Tunable
Monochtrmator	Diffracted beam	Diffracted beam (incident beam)	Incident beam (double crystal)
peak-to-bkg ratio	Not good	Good	Very good
Counting time	Long (~ 1day)	Shorter (5-6 h)	Short (4-5 Mins)
Resolution (FWHM)	0.1-0.2	0.04-0.07	0.02-0.03
Detection limit	2-3 wt %	0.1 wt %	<0.1 wt %
Unit cell parameters	Ambiguous	Possible with more accuracy	Very accurate
Modulation	Below detection	Can be detected	Can be detected
Symmetry	Ambiguous	Possible	Possible
Crystal structure refinement	Possible	Possible	Possible
Crystal structure solution	Not possible	Possible	Possible

Typical XRD pattern of crystalline materials





Unit cell parameters of compounds

Determination of unit cell parameters

INDEXING

(Assignment of h k l to observed reflections)

CELL REDUCTION

(Search for other possibility of the unit cell)

REFINEMENT

(Minimization of errors)

Unit cell parameters of compounds

$$\frac{1}{d_{hkl}^{2}} = \frac{\lambda^{2}}{4 \times Sin^{2} \theta_{hkl}}$$
 (From Bragg's Law)

$$\frac{1}{d^2} = \frac{\frac{h^2}{a^2} Sin^2 \alpha + \frac{k^2}{b^2} Sin^2 \beta + \frac{l^2}{c^2} Sin^2 \gamma + \frac{2hk}{ab} (Cos\alpha \cdot Cos\beta - Cos\gamma) + \frac{2kl}{bc} (Cos\beta \cdot Cos\gamma - Cos\alpha) + \frac{2lh}{ca} (Cos\gamma \cdot Cos\alpha - Cos\beta)}{1 - Cos^2 \alpha - Cos^2 \beta - Cos^2 \gamma + 2Cos\alpha \cdot Cos\beta \cdot Cos\gamma}$$

$$\frac{1}{d^2} = h^2 (a^*)^2 + k^2 (b^*)^2 + l^2 (c^*)^2 + 2hka^* b^* \cos\gamma^* + 2klb^* c^* \cos\alpha^* + 2lhc^* a^* \cos\beta^*$$

$$V^{2} = a^{2}b^{2}c^{2}\left(1 - \cos^{2}\alpha - \cos^{2}\beta - \cos^{2}\gamma + 2\cos\alpha \cdot \cos\beta \cdot \cos\gamma\right)$$

(h, k, l are integers, called as Miller Indices)

Unit cell parameters of compounds

Cubic System

Tetragonal System

$$\frac{1}{d_{hkl}^{2}} = \frac{h^{2} + k^{2} + l^{2}}{a^{2}}$$

$$\frac{1}{d_{hkl}^{2}} = \frac{h^{2} + k^{2}}{a^{2}} + \frac{l^{2}}{c^{2}}$$

$$a^{2} = (h^{2} + k^{2} + l^{2}) \times d_{hkl}^{2}$$

$$a^{2} = (h^{2} + k^{2}) \times d_{hk0}^{2}$$

$$c^{2} = (l^{2}) \times d_{00l}^{2}$$

$$c^{2} = (h^{2} + k^{2} + l^{2}) \times d_{hkl}^{2} - (h^{2} + k^{2}) \times d_{hk0}^{2}$$

Computer Programs for unit cell determination

TREOR, VISER, ITO, CELL, UNITCELL, POWDER, INDEXING

SrTiO3 Test P Sys.CU Lo20/L	DF set BIC c 23 M20	P =999.0 A	= 389.	Lamb 12	oda=	1.	5406	500 (<mark>F2(</mark>)=999.0(0.000, 2	<mark>20)</mark> X	20= 0
a= 5.9 Line 0. C.	d-spac obs.	ing A. calc.	Int. obs.	wt.	Inc h	dice k	es 1	SinSqTh obs.	neta*E4 calc.	2Tł obs.	neta De calc.	g. diff
1 1 2 2 3 3 4 4 5 5 6 6 7 7 8 8	3.9050 2.7613 2.2545 1.9525 1.7463 1.5942 1.3806 1.3017	3.9050 2.7613 2.2546 1.9525 1.7464 1.5942 1.3806 1.3017		1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0	1 1 2 2 2 2 3 2	0 1 1 0 1 1 2 0 2	0 0 1 0 1 0 0 1 0 0	389.1 778.2 1167.3 1556.5 1945.6 2334.7 3112.9 3502.0	389.1 778.2 1167.3 1556.5 1945.6 2334.7 3112.9 3502.0	22.75 32.40 39.96 46.47 52.35 57.79 67.83 72.57	22.75 32.40 39.96 46.47 52.35 57.79 67.83 72.57	$\begin{array}{c} 0.000\\ 0.000\\ 0.000\\ 0.000\\ 0.001\\ 0.000\\ 0.000\\ 0.000\\ 0.000\\ \end{array}$
9 9 10 10 11 11 12 12 13 13 14 14 15 15 16 16	1.2349 1.1774 1.1273 1.0831 1.0437 0.9762 0.9471 0.9204	1.2349 1.1774 1.1273 1.0831 1.0437 0.9762 0.9471 0.9204		1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0	3 2 3 3 4 3 4 3 4 3	1 2 2 2 0 2 1 3	0 1 2 0 1 0 2 0 0	3891.1 4280.3 4669.3 5058.5 5447.6 6225.8 6615.0 7004 0	3891.2 4280.3 4669.4 5058.5 5447.6 6225.8 6615.0 7004 1	77.19 81.72 86.21 90.67 95.14 104.19 108.84	77.19 81.72 86.21 90.67 95.14 104.19 108.84	0.000 0.000 0.000 -0.001 0.000 0.000
17 17 18 18 19 19 20 20 21 22	0.8959 0.8732 0.8521 0.8325	0.8959 0.8732 0.8521 0.8325 0.7971 0.7810		1.0 1.0 1.0 1.0 1.0	4 3 4 3 4 3 4 5 4	1 3 2 2 3 2 0 3	1 1 0 1 2 2 0 0	7393.2 7782.4 8171.4 8560.5	7393.2 7782.3 8171.4 8560.5 9338.8 9727.9	118.60 123.81 129.37 135.41	118.60 123.81 129.37 135.41 150.20 161.01	0.000 0.001 0.000 -0.001
FINISH												

POWD-OU.1

Conditions L	attice centering	Conditions	Symmetry elem.
		hko	
h + k = 2n	С	h = 2n	a-glide
k + l = 2n	Α	k = 2n	b-glide
l + h = 2n	В	h+k = 2n	n-glide
h+k, k + I and h+	l <i>=2n</i> F	h+k = 4n (h,k = 2n)	d-glide
h + k + l = 2n	I	hoo	
-h+k+l = 3n	R	h = 2n	2 ₁ , 4 ₂ along <100>
h-k+l = 3n	R	h = 4n	4 ₁ , 4 ₃ along <100>
No condition	Ρ	ool $I = 2n 2_1, 4$ $I = 3n 3_1, 3$ I = 6n 6 6	$B_{1}, 6_{3} \text{ along } <001>$ $B_{2}, 6_{2}, 6_{4} \text{ along } <001>$

Structure refinement from Powder XRD data (Rietveld method)

Rietveld Analysis is based on

- a. Optimization of Profile parameters Suitable profile function defined to construct the peak
- **b. Optimization of Structural parameters** Model structure (Space group, unit cell parameters, Position coordinates) are essential



a. Profile parameters

1. Background

* Can be selected by interpolation of selected points

* Can be modeled with polynomial function

2. Peak Profile

Profile is defined with specific function, like

- * Gaussian
- * Lorentzian
- * Combination as Pseudo Voigt function
- * Cauchy

...etc.

$$H_{hkl}^2 = U \tan^2 \vartheta + V \tan \vartheta + W$$

3. Preferred Orientation

The preferred Orientation need to avoided as far as possible The sample nature may some time force orientation 4. Asymmetry Asymmetry of the peak shape 5. Displacement, Transparencies, Two theta zero Lead to the peak shift and accurate peak positioning Experimental and instrumental 6. Lorenz and Polarization Correction

7. Size and strain factors

b. Structural parameters

- 1. Chemical details
- 2. Scattering factor/length of various atoms
- 3. Unit cell parameters and space group
- 4. Positional details of all atoms
- 5. Occupancies
- 6. Thermal parameters (*if available*)

Structure factor calculations

N

$$F_{hkl} = \sum_{j \to l}^{j \to N} f_j e^{2\pi i(hx_j + ky_j + lz_j)}$$

Where

- F_{hkl} : Amplitude of scattered radiation from the plane hkl
- f_j : Scattering factor of the atom j at the diffraction angle θ
- (x_{j}, y_{j}, z_{j}) : Fractional coordinates of the atom j in the unit cell
 - : Number of atoms in the unit cell

$$f = f_0 e^{\frac{-B\sin^2\vartheta}{\lambda^2}}$$

- f_0 : Scattering factor of an atom when it is rest and at 0°
- λ : Wavelength of x-ray
- **θ** : Angle of diffraction
- **B** : Isotropic temperature factor

B = $8\pi^2 u^2$, where u^2 = mean of square displacement of the atom

(The exponential term is called Debye-Waller factor)

Intensity calculation

where

$$Y_{ci} = y_{bi} + s \sum_{hkl} L \times P \times n \times |F_{hkl}|^2 \phi(2\vartheta_i - 2\vartheta_{hkl}) \times P_{hkl} \times A$$

Y _{ci}	: Calculated intensity at the i _{th} step
y _{bi}	: Background intensity at i _{th} step
L	: Lorenz factor
Р	: Polarization factor
n	: Multiplicity
$/F_{hkl}/^2$: Structure factor for hkl reflections
$\varphi(2\theta_i - 2\theta_{hkl})$: Profile function
P_{hkl}	: Preferred orientation function
A	: Absorption correction
S	: scale factor

Error calculation and minimization

$$D = \sum_{i=1}^{n} w_{i} (Y_{io} - Y_{ic})^{2}$$

the quantity D (residual) is minimized in the least square refinements

Where Yio :	Intensity observed at i th step
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- Yic : Intensity calculated at ith step
- wi : weighting factor and usually $1/Y_{oi}$

the model structure updated is applied shift $\Delta \xi$ in each step to reduce the error

Judgment of refinements

Difference plot and Residual indicator (R-Value)

Residual indicator (R-Value)

$$R_{p} = \frac{\sum (Y_{io} - Y_{ic})}{\sum Y_{ic}}$$

$$R_{wp} = \left[\frac{\sum w_{i}(Y_{io} - Y_{ic})^{2}}{\sum w_{i}Y_{io}^{2}}\right]^{1/2}$$

$$R_{exp} = \left[\frac{N - P + C}{\sum w_{i}Y_{io}^{2}}\right]^{1/2}$$

$$R_{F} = \frac{\sum \left|I_{hkl(o)}^{1/2} - I_{hkl(c)}^{1/2}\right|}{\sum I_{hkl(o)}^{1/2}}$$

$$R_{B} = \frac{\sum \left|I_{hkl(o)} - I_{hkl(c)}\right|}{\sum I_{hkl(o)}}$$

$$S = \frac{W_{wp}}{W exp} = \chi$$

$$d = \frac{\sum ((\Delta_{i} / \sigma_{i}) - (\Delta_{i-1} / \sigma_{i-1}))^{2}}{\sum (\Delta_{i} / \sigma_{i})^{2}}$$

R. pattern

R. weighted Pattern

R. expected

R. structure factor

R. Bragg

Goodness of fit

Durbin – Watson statistics

Computer Programs for Rietveld refinements

FullProf, GSAS, Rietan, DBWS, etc.

Do

Try and get success

Try till no other possible solution

Keep tab on correlated parameter

Check the chemical and physical sensibility of the refined results

Verify if possible

Convince yourself

Don't

Never try with bad data and bad structural model













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A_2(MoO_4)_3, A = Al<sup>3+</sup>, Cr<sup>3+</sup>, Fe<sup>3+</sup>
AMo<sub>2</sub>O<sub>7</sub> (A = Zr<sup>4+</sup> and Hf<sup>4+</sup>)
A_2(WO_4)_3, A = Al<sup>3+</sup>, Nd<sup>3+</sup> and Y<sup>3+</sup>
KScMo<sub>2</sub>O<sub>8</sub>, KAIMo<sub>2</sub>O<sub>8</sub>
AIPO<sub>4</sub>, GaPO<sub>4</sub>, BPO<sub>4</sub>
VP<sub>2</sub>O<sub>7</sub>
NbOPO<sub>4</sub>
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HT-XRD STUDIES ON Al_{1-x}Ga_xPO₄ (cristo. type)



HT-XRD STUDIES ON BPO₄ (Tetragonal, cristobalite type) Tetragonal (space group I-4. No. 82)

a = 4.3447(2), c = 6.6415(5) Å V = 125.37(1) Å^{3,} Z = 2.

B: 2c (0,1/2,1/4); P: 2a (0,0,0); O: 8g (x,y,z)



Variation thermal expansion coefficients with inter-polyhedral angle of *Cristobalite* type compounds

 $\alpha_{\rm V}$ (/°C) = -191.32 + 4.33 × [θ]

- $0.02 \times [\theta]^2$

- **1.** AlPO₄ (at 300°C)
- 2. AIPO₄ (at 300°C) (*lit. data*)
- **3.** SiO₂ (at 300°C) (*lit. data*)
- 4. $Al_{0.8}Ga_{0.2}PO_4$ (300°C)
- 5. $Al_{0.5}Ga_{0.5}PO_4$ (400°C)
- 6. Al_{0.2}Ga_{0.8}PO₄ (600°C)
- 7. GaPO₄ (700°C)

8. BPO₄ (25°C)



Variation of structural parameters of BPO₄ with temperature



Transformation topology for cristobalite frame with Temp. /Press.

Perovskites...

Multiferroic Ba and Mn co-doped BiFeO₃

Preparation: Xerogel method

Magnetization with Field

Electric polarization with Field

Rietveld plots for XRD data

Summary of structure, electrical and magnetic properties

Sample	Symmetry	Hc	Mr	Ps
		(kOe)	(emu/g)	(µC/cm ²)
BiFeO ₃	R3c	0.4	0.005	3.8
BiFe _{0.8} Mn _{0.2} O ₃	R3c	0.7	0.3	3.0
Bi _{0.9} Ba _{0.1} Fe _{0.8} Mn _{0.2} O ₃	R3c, P4mm (87:13)	4.5	3.8	4.5
Bi _{0.9} Ba _{0.1} FeO ₃	P4mm	3.5	1.2	0.25

A new elpasolite-type (NH₄,K)₃VO₂F₄

Low temperature solid state reaction of $KVO_3 + NH_4HF_2$

Orthorhombic (Space Group: Immm, No. 71) a = 8.9584(4), b = 18.6910(14), c = 6.2174(4) Å, V = 1041.04(11) Å³, Z = 6, Rp: 10.9, Rwp: 14.1, χ 2: 3.77, R_B: 12.0 Second Phase: K₂VO₂F₃ (Orthorhombic, Pnma, No. 62) Fraction: 7.8(4) wt %

Position coordinates of $(NH_4, K)_3 VO_2 F_4$

Atoms	Wyc	х	У	Z	B(Ų)	Occ.
V1	2a	0	0	0	1.4(3)	1
V2	4 g	0	0.6829(4)	0	3.3(2)	1
(K/N)1	4h	0	0.1402(6)	0.5	6.2	0.80(1) 0.20(1)
(K,N)2	8n	0.2836(11)	0.1679(7)	0	5.4	0.47(2), 0.53(2)
(K,N)3	4f	0.25	0.5	0	6.8	0.14(2), 0.86(2)
(K,N)4	2b	0	0.5	0.5	5.1	0.32(1), 0.68(1)
01	81	0	0.7346(7)	0.780(3)	4.8	1
02	8n	0.184(1)	0.6777(9)	0	4.8	1
03	81	0	0.6042(7)	0.789(2)	4.8	1
04	4g	0	0.9098(6)	0	4.8	1
05	4e	0.763(2)	0	0	4.8	1
06	41	0	0	0.726(2)	4.8	1

Structural composition $(NH_4)_{1.7}K_{1.3}V(OF)_6$

Phase transition and thermal stability

Reversible structural transition at 343 K

Weight loss (30%) between 300 to 623 K is close to that expected (26%) for the loss of 1.7 NH4F

Agrees with structural composition $(NH_4)_{1.7}K_{1.3}V(OF)_6$.

Ferroelectric properties

Electric Field, V/cm

Proton conduction properties

Dilute magnetic semiconductor

- ZnO-Fe
- ZnO-Co
- ZnO-Ni
- In₂O₃-Fe

Powder XRD data of In₂O₃-10 % Fe

Better signal to noise ratio and peak shape in shorter time

	10 %	Fe In ₂ O ₃
Source	Sealed tube (CuKa)	Rotating anode (CuKα)
Operation power	1.2 KW	6 KW
Space group	Ia-3	Ia-3
a (Å)	10.0510(11)	10.0511(6)
V (Å) ³	1015.4(2)	1015.4(1)
Inl (8b) (¼,¼,¼)		
In2 (24d) $(x, \theta, \frac{1}{2})$	-0.0300(2)	-0.0323(1)
O(48e)(x,y,z)	0.3862(16)	0.3929(7)
	0.1625(10)	0.1566(7)
	0.3823(19)	0.3864(9)
R _{ap}	6.87	6.82
R _{ap}	9.11	9.52
X ²	1.32	1.49
R ₈	2.25	1.98

Rietveld refinement plot of x-ray data collected on rotating anode source

Comparison of structural parameters of two sources

Oxidation of Ce₂Zr₂O₇

Powder neutron diffraction studies

Parameters	Compositions						
Molecular formula	Ce ₂ Zr ₂ O ₇	Ce ₂ Zr ₂ O _{7.52}	Ce ₂ Zr ₂ O ₈				
Color	Black	Gray	Bright yellow				
Crystal system	Cubic	Cubic	Cubic				
Space group	Fd3m (No. 227)	F-43m (No. 216)	P2 ₁ 3 (No. 198)				
a (Å)	10.6924(3)	10.6199(2)	10.5443(2)				
V (Å ³)	1222.43(11)	1197.74(6)	1172.34(6)				
Density (cal)	6.245 g/cc	6.466 g/cc	6.693 g/cc				
R _p , R _{wp}	0.0714, 0.0533	0.0597, 0.0447	0.0664, 0.0500				
χ^2	3.972	2.928	3.947				
R _F ²	0.0717	0.0421	0.0519				

Finally

- Data quality has a significant role of the accurate structural parameters.
- The structure is never complete unless it is verified from single crystal data or refined from high resolution synchrotron data.
- The structure is never accurate if it failed to explain the properties.
- Though Rietveld analysis has large a number of limitation, so far this is the accepted method for ceramics oxides.

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